Smart Mater. Struct. **14** (2005) 850–861 doi:10.1088/0964-1726/14/4/045

Parametric identification of asymmetric buildings from earthquake response records

C C Lin[1,2,4], L L Hong[3], J M Ueng[2], K C Wu[2] and C E Wang[2]

[1] College of Engineering, National Chung Hsing University, Taichung 40227, Taiwan, Republic of China
[2] Department of Civil Engineering, National Chung Hsing University, Taichung 40227, Taiwan, Republic of China
[3] Department of Civil Engineering, National Cheng-Kung University, Tainan 701, Taiwan, Republic of China

E-mail: cclin3@dragon.nchu.edu.tw

Received 22 December 2003, in final form 27 May 2005
Published 28 July 2005
Online at stacks.iop.org/SMS/14/850

Abstract
This paper aims to evaluate the modal frequencies, damping ratios and mode shapes of an asymmetric building, modeled as general torsionally coupled buildings using a modified random decrement method together with the Ibrahim time domain technique based only on few floor acceleration response records from earthquakes. It is not necessary to measure earthquake excitation input. The general relationship between the reduced random decrement signature and the true free vibration response is derived analytically. Because only partial floor response measurements are used, a mode shape interpolation technique is developed to estimate the mode shape values for the locations without measurement, such that all floor responses can be obtained. The results were obtained from simulation data from a five-story building under the 1940 El Centro earthquake and actual records from a seven-story RC school building in north-eastern Taiwan, due to an earthquake near the building site. The results show that the proposed system identification technique is capable of identifying structural dominant modal parameters and responses accurately even with highly coupled modes and high levels of noise contamination.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

To better understand the dynamic behavior of structures during environmental loads like strong winds or earthquakes, many existing buildings are installed with instruments like accelerometers. The measurements from these sensors, even caused by micro-tremors, are very informative and useful for the identification of structural dynamic parameters (e.g. modal frequency, damping ratio, and mode shape), which has become an intense research subject in recent years (Zaslavsky and Shapira 1997). The identified dynamic parameters can be used to evaluate structural damage and predict structural response to a future, different excitation. In structural control, the control device design requires knowledge of the system parameters of the controlled structure. Thus, it is important that system identification be carried out in conjunction with structural control.

A general building with a nominally symmetrical plan is usually associated with some degree of coupling between the translational and torsional vibration due to the non-coincidence of the mass and stiffness centers. This building will undergo lateral as well as torsional vibrations simultaneously even when subjected to purely translational excitations. As a result, each floor vibration will differ from that of the building, which has a symmetrical plan and responds only in planar vibrations.

[4] Author to whom any correspondence should be addressed.

0964-1726/05/040850+12$30.00

Neglecting torsional vibration may lead to underestimating the structural responses (Hejal and Chopra 1989). Therefore, it is more appropriate and essential to model a structure with a torsional degree of freedom.

Identifying a torsionally coupled building system from dynamic measurements has rarely been proposed in the literature (Kozin and Natke 1986, Luz and Wallaschek 1992). Most of the papers presented discussed torsionally coupled building systems only for planar frame structures (Toki *et al* 1989, Benzoni and Gentile 1993, Mau and Aruna 1994, Wang and Haldar 1994) or identified two translational and one torsional modal parameter separately (Kadakal and Yüzügüllü 1996, Torkamani and Ahmadi 1988). Further, traditional system identification techniques require the full input excitation measurement and its corresponding responses. However, the input excitation is generally difficult to define and accurately measure. Moreover, a real structure usually possesses a large number of degrees of freedom. It is impossible and impractical to acquire full measurements because of the limited number of sensors. Thus, system identification based only on response measurements at a few degrees of freedom becomes necessary from a practical point of view.

The random decrement (randomdec) method, originally developed by Cole (1971, 1973) for single measurements to detect damage in aerospace structures, was commonly used for the identification of modal damping when only the response data under random excitations were available. Owing to its efficiency and simplicity in processing vibration measurements and the lack of requirement for input excitation measurements, this method is applied extensively to detect damage in civil structures (Yang and Caldwell 1976) and offshore structures (Yang *et al* 1984). The identification of structural parameters from time history responses has been investigated by many researchers (Caravani *et al* 1977, Shinozuka and Yun 1982, Mickleborough and Pi 1989). Most of the available identification techniques are mathematically complicated and sensitive to noise. Few techniques address the most critical factors such as (1) the number and the location of measurements, (2) the direction of measurements, (3) the mode coupling, and (4) the number of modes for the response and noise levels as related to the number of degrees of freedom allowed in the identification model. Among these methods, the Ibrahim time domain (ITD) technique (Ibrahim and Mikulcik 1977, Ibrahim 1986) was studied most extensively and generally accepted as the approach to solving the problem of noise contamination and the number of measurements. However, the ITD technique is only applicable to free response data and emphasizes the identification of modal frequencies and damping ratios. In the case of partial measurements, only the mode shape values corresponding to the instrumented degrees of freedom could be obtained. To estimate the dynamic responses for the locations without measurements, the complete mode shapes should be found.

In this study, a system identification technique was developed to evaluate the modal frequencies, damping ratios and mode shapes of general torsionally coupled buildings based on only few floor absolute acceleration responses induced by earthquakes. It is not necessary to measure the input earthquake excitation. A modified random decrement method,

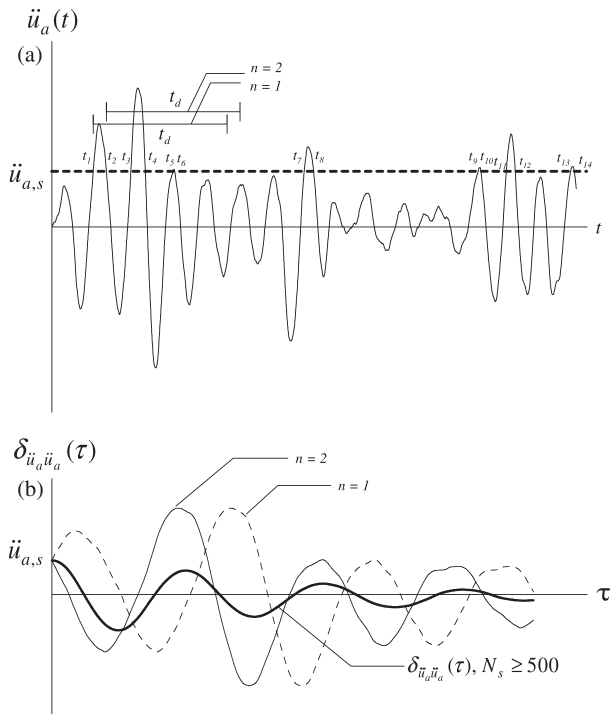

Figure 1. (a) Response measurement and crossing times. (b) Extraction of free decay signature from response measurement.

which the response record length is first extended and considers the correlation among measurements, is employed to reduce the extended response data to extract their corresponding free vibration responses. The mathematical basis, which shows the reduced random decrement acceleration signature, equal to the true free vibration response, is derived. Finally, the ITD technique is applied to calculate the structural modal frequencies, damping ratios, and mode shapes. Because partial floor response measurements are used, only the mode shape values corresponding to the instrumented degrees of freedom could be obtained. A mode shape interpolation technique was developed in this study to estimate the mode shape values for the locations without measurement, such that each floor response is estimated. The results were obtained from simulation data from a five-story building under the 1940 El Centro earthquake and actual records from a seven-story reinforced concrete (RC) building in north-eastern Taiwan due to the 1994 Nan-Au earthquake ($M_L = 6.2$). The results show that the proposed system identification technique is capable of identifying structural dominant modal parameters and responses accurately even with highly coupled modes and a high level of noise contamination. The small number of response measurements and simple on-line calculations make the proposed method favorable to real implementation.

2. Modified random decrement method

As stated, in 1971, Cole developed the random decrement method to estimate structural damping and assess damage in aerospace structures. It was based only on physical intuition rather than rigorous mathematical derivation. From 1982, Vandiver *et al* and further Bedewi (1986) derived a general

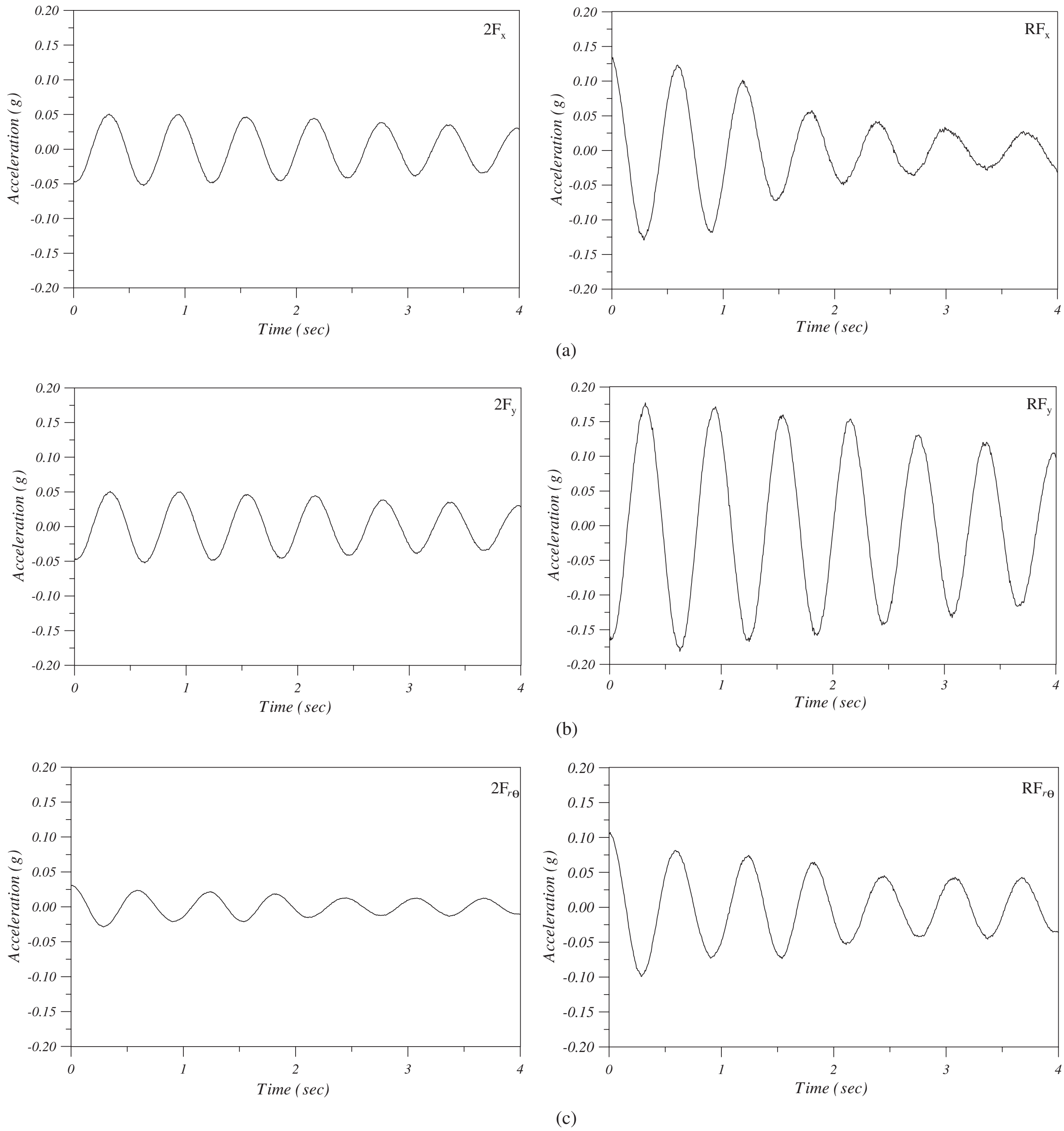

Figure 2. (a) Free decay signatures at second floor and roof level in the x direction. (b) Free decay signatures at second floor and roof level in the y direction. (c) Free decay rotation signatures at second floor and roof level.

mathematical basis for the randomdec technique to single-degree-of-freedom (DOF) and multiple-DOF time-invariant linear systems. They showed that if a time function $u(t)$ is a zero-mean, stationary, Gaussian random process, its randomdec signature, $\delta_{uu}(\tau)$, is expressed as

$$\delta_{uu}(\tau) = \frac{R_{uu}(\tau)}{R_{uu}(0)} u_{\mathrm{s}} \tag{1}$$

where u_{s} is a constant (but not zero) for extracting the randomdec signature of $u(t)$, and $R_{uu}(\tau)$ denotes the auto-correlation function of $u(t)$. In addition, they also proved that the randomdec displacement signature was equivalent to the free decay displacement response of the system with zero initial velocity and a constant initial displacement determined from the extraction process such as u_{s}. Since many existing buildings and bridges in the world are installed with accelerometers, acceleration data are usually measured when subjected to earthquake or other support excitations. It is necessary to find out and prove mathematically the equivalent relationship between the randomdec signature and the free decay acceleration response of structures.

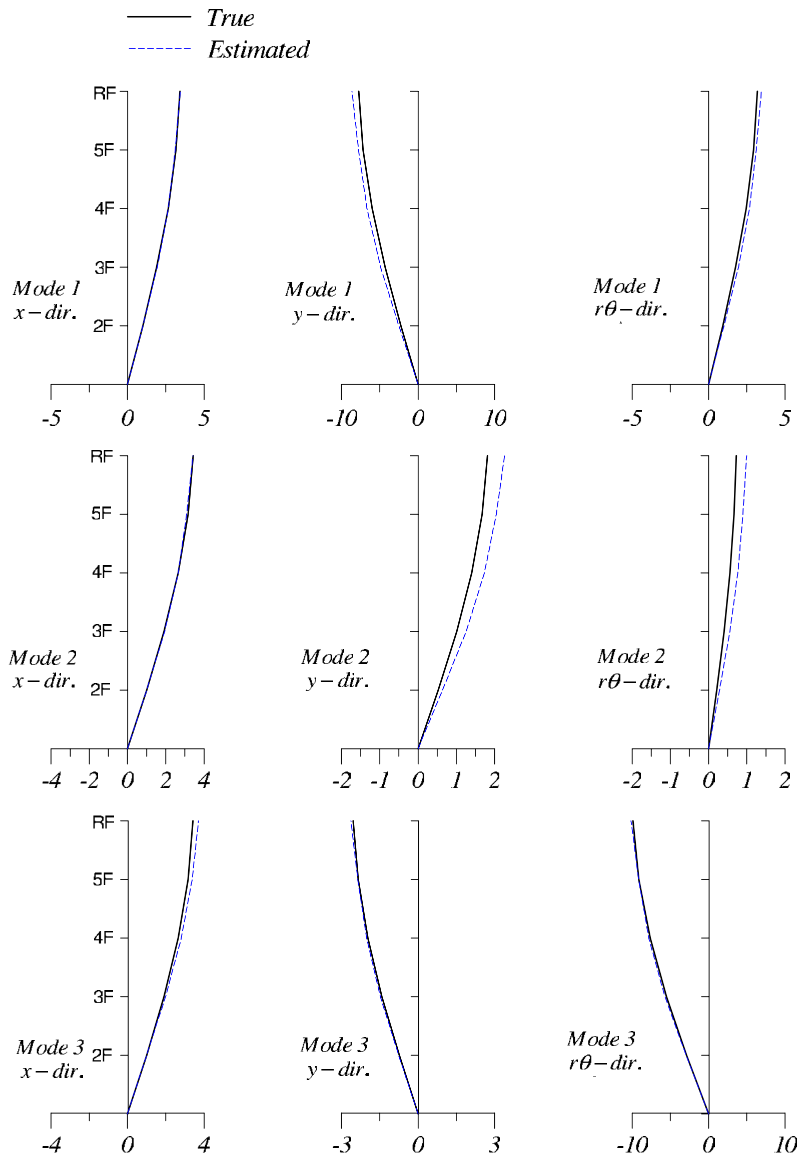

Figure 3. Identified versus true mode shapes of the five-story building.

For a single DOF time-invariant, linear system with undamped natural frequency, ω, and damping ratio, ξ, its free decay displacement, velocity, and acceleration responses relative to the base are written as

$$u(t) = \mathrm{e}^{-\xi\omega t}\left\{u(0)\cos(\omega_{\mathrm{d}}t) + \frac{1}{\omega_{\mathrm{d}}}[\xi\omega u(0) + \dot{u}(0)]\sin(\omega_{\mathrm{d}}t)\right\} \tag{2}$$

$$\dot{u}(t) = \mathrm{e}^{-\xi\omega t}\left\{\dot{u}(0)\cos(\omega_{\mathrm{d}}t) + \frac{1}{\omega_{\mathrm{d}}}[-\omega^2 u(0) - \xi\omega\dot{u}(0)]\sin(\omega_{\mathrm{d}}t)\right\} \tag{3}$$

$$\ddot{u}(t) = \mathrm{e}^{-\xi\omega t}\left\{[-2\xi\omega\dot{u}(0) - \omega^2 u(0)]\cos(\omega_{\mathrm{d}}t) + \left[\left(\xi\omega\omega_{\mathrm{d}} + \frac{\xi^3\omega^3}{\omega_{\mathrm{d}}}\right)u(0) + \left(\frac{\xi^2\omega^2}{\omega_{\mathrm{d}}} - \omega_{\mathrm{d}}\right)\dot{u}(0)\right]\sin(\omega_{\mathrm{d}}t)\right\} \tag{4}$$

where $u(0)$ and $\dot{u}(0)$ are the initial displacement and velocity of the structure, respectively, and ω_{d} is the damped natural frequency. According to equation (1), their own randomdec signatures take the forms

$$\delta_{uu}(\tau) = u_{\mathrm{s}}\mathrm{e}^{-\xi\omega\tau}\left[\cos(\omega_{\mathrm{d}}\tau) + \frac{\xi}{\sqrt{1-\xi^2}}\sin(\omega_{\mathrm{d}}\tau)\right] \tag{5}$$

$$\delta_{\dot{u}\dot{u}}(\tau) = \dot{u}_{\mathrm{s}}\mathrm{e}^{-\xi\omega\tau}\left[\cos(\omega_{\mathrm{d}}\tau) - \frac{\xi}{\sqrt{1-\xi^2}}\sin(\omega_{\mathrm{d}}\tau)\right] \tag{6}$$

$$\delta_{\ddot{u}\ddot{u}}(\tau) = \frac{\ddot{u}_{\mathrm{s}}}{\frac{\omega}{4\xi}(4\xi^2-1) + \hat{\delta}(0)}\left\{\frac{\omega}{4\xi}\mathrm{e}^{-\xi\omega\tau}\left[(1-4\xi^2)\cos(\omega_{\mathrm{d}}\tau) + \frac{\xi(4\xi^2-3)}{\sqrt{1-\xi^2}}\sin(\omega_{\mathrm{d}}\tau)\right] + \hat{\delta}(\tau)\right\} \tag{7}$$

where u_{s}, $\dot{u}_{\mathrm{s}}$, and $\ddot{u}_{\mathrm{s}}$ are constant values of relative displacement, velocity, and acceleration, respectively, and $\hat{\delta}(-)$ denotes the Dirac delta function. Comparing equations (2)–(4) with (5)–(7), it is found that the displacement randomdec signature is equivalent to its free decay response with $u(0) = u_s$ and $\dot{u}(0) = 0$. This conclusion also holds for velocity response with $u(0) = 0$ and $\dot{u}(0) = \dot{u}_s$, but is not true for acceleration response. The above studies dealt with the relative responses of the structure with respect to its fixed base. For the case of base (or say support) excitation (e.g. earthquake), the free decay absolute acceleration response which is usually measured takes the form

$$\ddot{u}_a(t) = e^{-\xi\omega t}\Bigg\{[-2\xi\omega\dot{u}(0) - \omega^2 u(0)]\cos(\omega_d t) + \left[\left(\frac{\xi\omega^3}{\omega_d}\right)u(0) + \left(\frac{\omega^2(2\xi-1)}{\omega_d}\right)\dot{u}(0)\right]\sin(\omega_d t)\Bigg\}. \quad (8)$$

Similarly, based on equation (1), the randomdec signature of absolute acceleration was derived and expressed as

$$\delta_{\ddot{u}_a\ddot{u}_a}(\tau) = \ddot{u}_{a,s}e^{-\xi\omega\tau}\left[\cos(\omega_d\tau) + \frac{\omega\xi(1-4\xi^2)}{(4\xi^2+1)\omega_d}\sin(\omega_d\tau)\right] \quad (9)$$

where $\ddot{u}_{a,s}$ is a constant value of absolute acceleration. From equations (8) and (9), it can be shown that $\delta_{\ddot{u}_a\ddot{u}_a}(t)$ in equation (9) is exactly the same as $\ddot{u}_a(t)$ with initial displacement and initial velocity

$$u(0) = -\frac{\ddot{u}_{a,s}}{\omega^2(4\xi^2+1)}; \qquad \dot{u}(0) = -\frac{2\ddot{u}_{a,s}\xi}{\omega(4\xi^2+1)}. \quad (10)$$

2.1. Extraction of randomdec signature

Let $\ddot{u}_a(t)$ be an absolute acceleration response measurement (with or without noise) at a certain location in a structure, as shown in figure 1(a), induced by zero-mean, stationary random support excitations. This time history is divided into short segments with a duration of t_d, which is several times the structural fundamental period. The random decrement method consists of the following analysis steps to obtain the free decay response.

(1) Calculate an amplitude $\ddot{u}_{a,s}$, which is usually the rms (root-mean-square) value of $\ddot{u}_a(t)$.

(2) Select the starting time t_i of each segment such that

$$\ddot{u}_a(t_i) = \ddot{u}_{a,s}, \qquad i = 1, 2, 3, \ldots$$
$$d\ddot{u}_a(t_i)/dt \geqslant 0, \qquad i = 1, 3, 5, \ldots;$$
$$\text{and} \qquad d\ddot{u}_a(t_i)/dt \leqslant 0, \qquad i = 2, 4, 6, \ldots.$$

(3) Average N_s segments of the response measurement to yield a time function, $\delta_{\ddot{u}_a\ddot{u}_a}(\tau)$, i.e.

$$\delta_{\ddot{u}_a\ddot{u}_a}(\tau) = \frac{1}{N_s}\sum_{i=1}^{N_s}\ddot{u}_a(t_i+\tau); \qquad 0 \leqslant \tau < t_d. \quad (11)$$

This is called the randomdec signature of absolute acceleration, as shown in figure 1(b). The unique form of this signature and no necessity to input excitation measurements make the random decrement method very attractive for application to system parameter identification and damage detection.

Basically, the original random decrement method was developed to process a single measurement. For multiple measurements taken from a real building, their correlation will be lost if the above analysis procedure is applied to each individual measurement independently. To overcome this problem, in this study, the crossing times t_i in step (2) were determined from one designated measurement. All measurements were then processed following step (3), simultaneously, to obtain their own free decay signatures. For building structures, it is suggested that lower (e.g. second) floor measurement be used because it contains more contents of high mode responses. Therefore, we can acquire enough segment numbers to superimpose into equation (11) in a shorter record length.

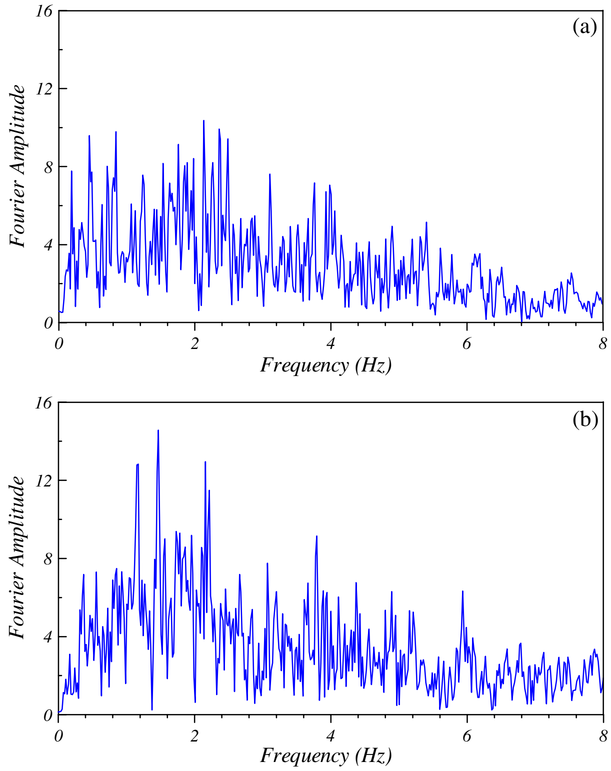

Figure 4. (a) Fourier amplitude spectrum of the 1940 El Centro earthquake in the EW direction. (b) Fourier amplitude spectrum of the 1940 El Centro earthquake in the NS direction.

2.2. Record length extension technique

From previous experience, to obtain the randomdec signature close to the true free decay response, N_s in equation (11) must be greater than 500. The greater the number of superpositions produced, the more accurate the obtained free decay response. Therefore, for a transient response record (e.g. earthquake response record), the record length should be extended.

In this paper, a record length extension technique is proposed based on the hypothesis that the earthquake input repeats at time instant t_j if $u(t_j)$ and $\dot{u}(t_j)$ are equal to the initial conditions $u(0)$ and $\dot{u}(0)$ simultaneously. For a record of length $n\delta t$ (δt is the sampling period), each superposition increases the length of $(j-1)\delta t$. In general, ten extensions are enough to obtain an accurate free decay response if an earthquake has a long strong motion. For an MDOF system, the time instant t_j will be decided so that the displacement and velocity responses of all DOFs are equal to their own initial values.

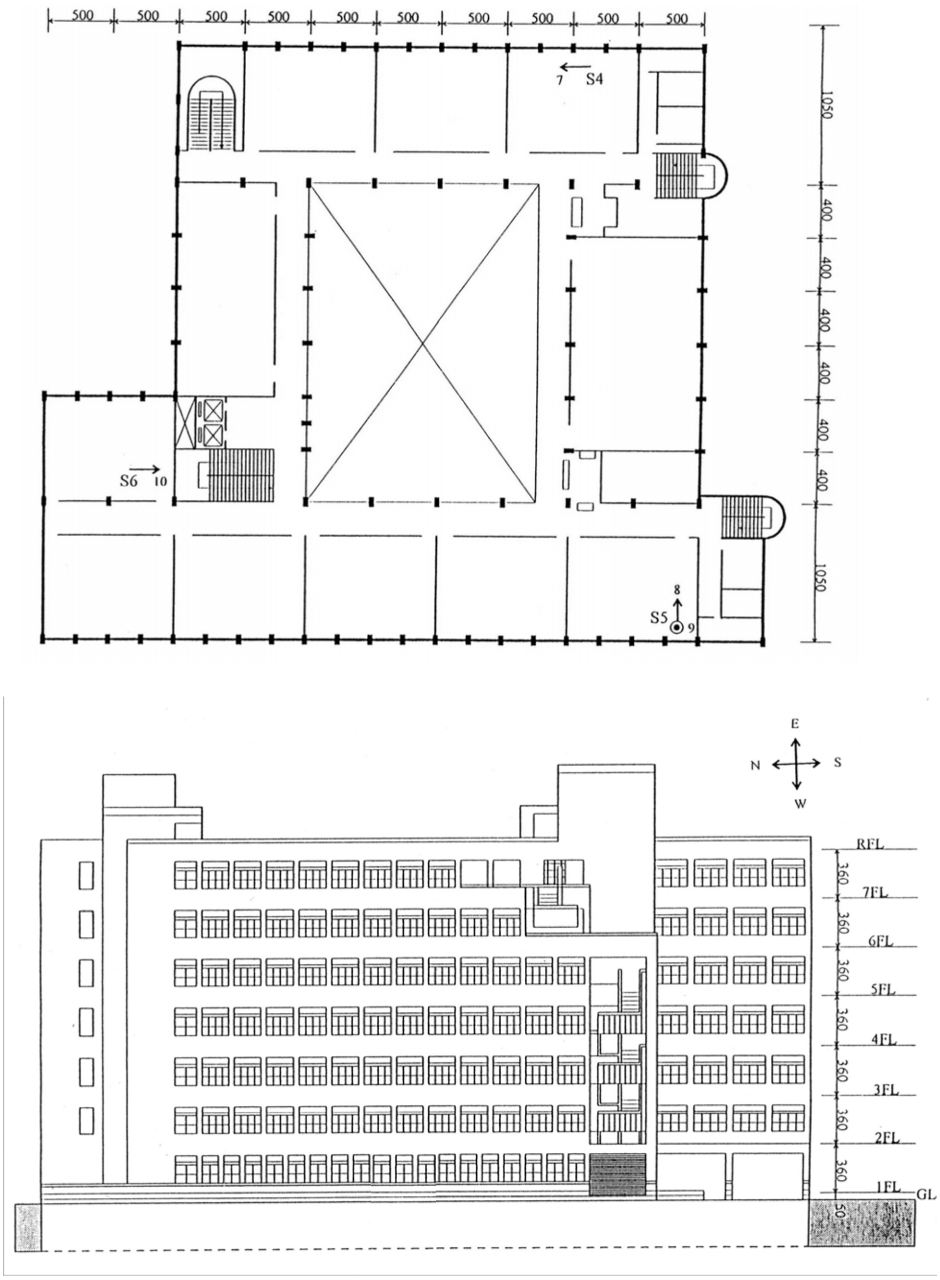

Figure 5. Typical plan and front view of the seven-story school building.

3. Ibrahim time domain technique

The free decay absolute acceleration responses at station l and time t_i can be expressed by the summation of m structural modes as

$$\delta_{\ddot{u}_a \ddot{u}_{a,l}}(t_i) = \ddot{x}_{l,i} = \sum_{k=1}^{2m} \lambda_k^2 \varphi_{lk} \mathrm{e}^{\lambda_k t_i} \tag{12}$$

where λ_k and φ_{lk} represent the kth complex eigenvalue and mode shape value at location l, respectively. The modal frequency ω_k and damping ratio ξ_k are then calculated by

$$\omega_k = |\lambda_k|; \qquad \xi_k = -\mathrm{Re}(\lambda_k)/\omega_k. \tag{13}$$

In equation (13), $\mathrm{Re}(\lambda_k)$ denotes the real part of λ_k. Suppose that the responses at n different stations are measured and m modal properties are desired to be identified. We may use any m measurements for s instants (when $n \geqslant m$) or repeat the available measurements (when $n < m$) to construct the response matrix $\ddot{\boldsymbol{X}}$ using time shifting schemes (Ibrahim and Pappa 1982) such that

$$\ddot{\boldsymbol{X}} = \Psi \lambda \Lambda \tag{14}$$

where

$$\ddot{\boldsymbol{X}} = \begin{bmatrix} \ddot{x}_{1,1} & \ddot{x}_{1,2} & \cdots & \ddot{x}_{1,s} \\ \ddot{x}_{2,1} & \ddot{x}_{2,2} & \cdots & \ddot{x}_{2,s} \\ \vdots & \vdots & \ddots & \vdots \\ \ddot{x}_{2m,1} & \ddot{x}_{2m,2} & \cdots & \ddot{x}_{2m,s} \end{bmatrix},$$

$$\Psi = \begin{bmatrix} \varphi_{1,1} & \varphi_{1,2} & \cdots & \varphi_{1,2m} \\ \varphi_{2,1} & \varphi_{2,2} & \cdots & \varphi_{2,2m} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_{2m,1} & \varphi_{2m,2} & \cdots & \varphi_{2m,2m} \end{bmatrix}$$

$$\lambda = \begin{bmatrix} \lambda_1^2 & 0 & \cdots & \cdots & 0 \\ 0 & \lambda_2^2 & 0 & \cdots & 0 \\ 0 & 0 & \lambda_3^2 & 0 & \vdots \\ \vdots & \vdots & \cdots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & \lambda_{2m}^2 \end{bmatrix},$$

$$\Lambda = \begin{bmatrix} e^{\lambda_1 t_1} & e^{\lambda_1 t_2} & \cdots & \cdots & e^{\lambda_1 t_s} \\ e^{\lambda_2 t_1} & e^{\lambda_2 t_2} & \cdots & \cdots & e^{\lambda_2 t_s} \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ e^{\lambda_{2m} t_1} & e^{\lambda_{2m} t_2} & \cdots & \cdots & e^{\lambda_{2m} t_s} \end{bmatrix}.$$

Similarly, the response matrix $\hat{\ddot{\boldsymbol{X}}}$ corresponding to the same measured stations and Δt later in time than those in equation (14) can be expressed as

$$\hat{\ddot{\boldsymbol{X}}} = \hat{\Psi}\lambda\Lambda. \tag{15}$$

The elimination of Λ and λ from equations (14) and (15) gives

$$\hat{\ddot{\boldsymbol{X}}} = \hat{\Psi}\Psi^{-1}\ddot{\boldsymbol{X}} = \boldsymbol{A}\ddot{\boldsymbol{X}} \tag{16}$$

and

$$\boldsymbol{A}\Psi = \Psi\alpha \tag{17}$$

where $\boldsymbol{A}$ is defined as the $(2m \times 2m)$ system matrix. Equation (16) is generally an over-determined system of simultaneous linear equations. The solution to obtain matrix $\boldsymbol{A}$ is not unique. Several approaches, such as the least square method and singular value decomposition, can be used. Moreover, equation (17) is a standard eigenvalue equation, which can be solved by any conventional method. The matrix α is a diagonal matrix with entries $\alpha_k = e^{\lambda_k \delta t}$. Let $\alpha_k = \beta_k + i\gamma_k$ and $\lambda_k = a_k + ib_k$ $(i = \sqrt{-1})$, then a_k and b_k are related to β_k and γ_k as

$$a_k = \frac{1}{2\Delta t}\ln(\beta_k^2 + \gamma_k^2); \qquad b_k = \frac{1}{\Delta t}\tan^{-1}\left(\frac{\gamma_k}{\beta_k}\right). \tag{18}$$

Once the eigenvalue λ_k is obtained, the kth modal frequency and damping ratio are calculated from equation (13). Based on the above derivations, it is also found that the eigenvectors Ψ of matrix $\boldsymbol{A}$ are the desired complex mode shapes of the structure.

4. Mode shape interpolation technique

As mentioned previously, the ITD method could estimate all desired modal frequencies and damping ratios. In the case of partial measurements, only the mode shape values at the instrumented degrees of freedom are identified. To obtain the complete mode shapes, in this paper, a mode shape interpolation method was developed to calculate the mode shape values for the locations without measurement.

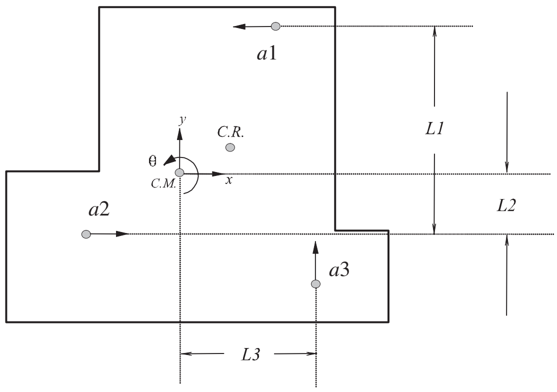

Figure 6. Relative position layout of the mass center and seismometers at the instrumented floors.

For an N-story torsionally coupled shear building structure (Kan and Chopra 1977), the jth mode shape Ψ_j is expressed as

$$\Psi_j = \begin{bmatrix} \phi_{xj} \\ \phi_{yj} \\ \phi_{\theta j} \end{bmatrix}, \qquad j = 1, 2, \ldots, 3N \tag{19}$$

where ϕ_{xj}, ϕ_{yj} and $\phi_{\theta j}$ denote its components in the x, y and θ directions. It is reasonably assumed that the ϕ_{xj}, ϕ_{yj} and $\phi_{\theta j}$ of any torsionally coupled building are the linear combination of the shear mode shapes, φ_j, of its corresponding N-story uncoupled (or say planar) system with the same mass distribution and uniform stiffness along the height. Then, we can form a set of functions $(\varphi_1 - \varphi_2), (\varphi_1 - \varphi_3), (\varphi_1 - \varphi_4), \ldots, (\varphi_1 - \varphi_N)$ as the basic ingredients for mode shape interpolation. Let p $(p \geqslant 2)$ be the number of floors where the vibration responses in x-, y- and θ-directions are measured. Then, the mode shape interpolation formulae for all $3N$ mode shapes in the d-direction $(d = x, y, \text{or } \theta)$ are expressed as

$$\phi_{d,j} = a_{d,j1}\varphi_1 + \sum_{b=2}^{p+\mathrm{int}((j-1)/3)} a_{d,jb}(\varphi_1 - \varphi_b); \quad j = 1, 2, \ldots, (N - p + 1) \times 3 \tag{20a}$$

$$\phi_{d,j} = a_{d,j1}\varphi_1 + \sum_{b=2}^{N} a_{d,jb}(\varphi_1 - \varphi_b); \quad j = (N - p + 1) \times 3 + 1, \ldots, 3N \tag{20b}$$

where $\mathrm{int}(-)$ indicates the integer part of the number in parenthesis. $a_{d,j1}$ and $a_{d,jb}$ are constant coefficients determined by the identified mode shape values and orthogonal conditions among modes. It has been verified that with any p $(p \geqslant 2)$ measured floors ($3p$ response measurements), there are always enough conditions to determine the unknown coefficients. More detailed illustrations can be found in the article by Ueng *et al* (2000).

It is noted that the proposed interpolation technique employs base vectors derived from a regular shear building. It may not be suitable for the following buildings:

(1) buildings with setback—this kind of buildings possess nonuniform stiffness, which violates the assumption of uniform stiffness along the height;

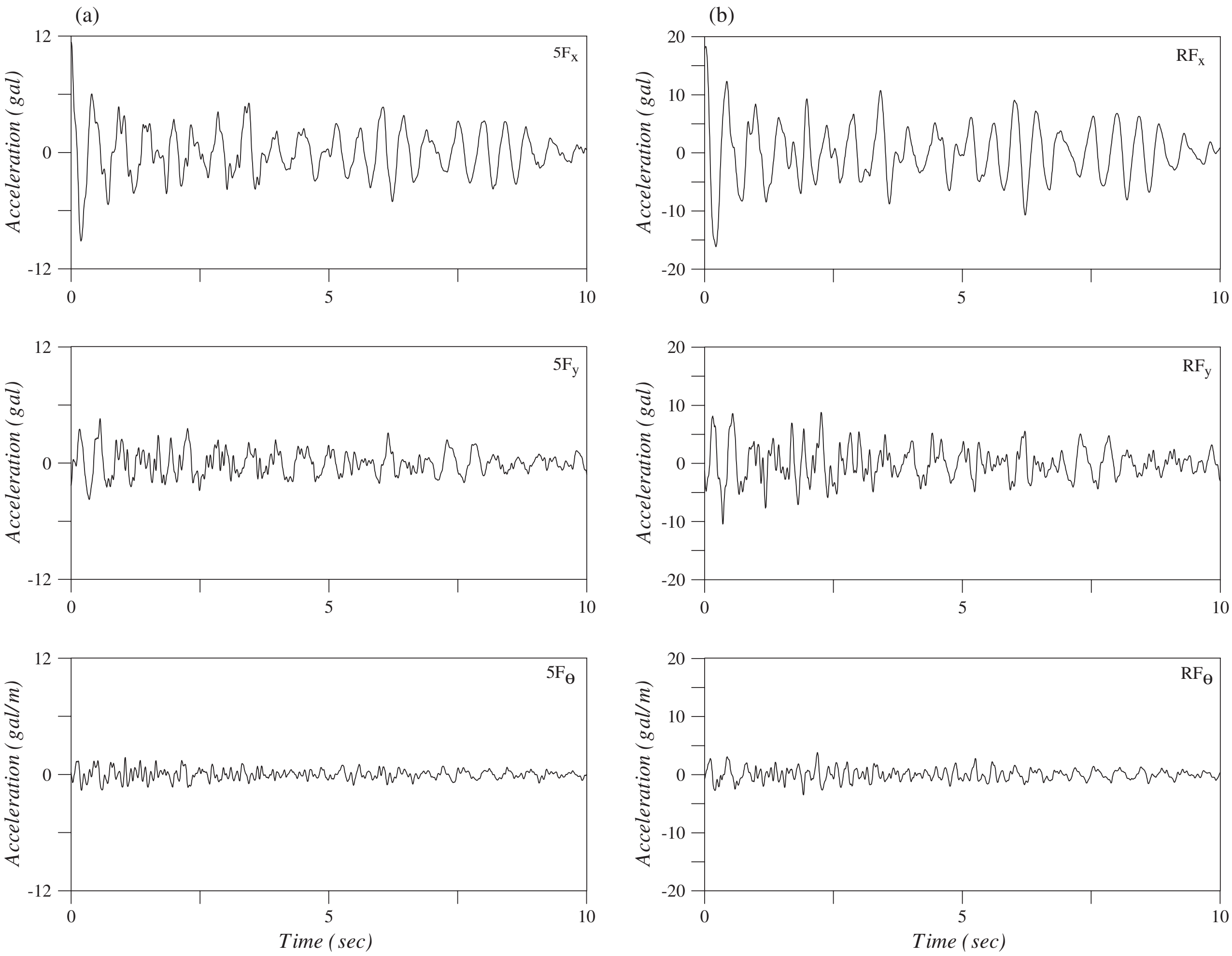

Figure 7. (a) Randomdec acceleration signatures at the mass center of the fifth floor of the seven-story school building. (b) Randomdec acceleration signatures at the mass center of the roof level of the seven-story school building.

Table 1. Physical system parameters of the five-story building.

Floor	Floor mass m (kg)	Story stiffness k_x (N m^{-1})	k_y (N m^{-1})	k_θ (N m)	Story eccentricity (m) e_x	e_y	Radius of gyration r (m)
1F	2.8×10^5	3.60×10^8	3.20×10^8	3.20×10^{10}	1.6	1.6	8.0
2F	2.6×10^5	3.55×10^8	3.15×10^8	3.15×10^{10}	1.6	1.6	8.0
3F	2.4×10^5	3.50×10^8	3.10×10^8	3.10×10^{10}	1.6	1.6	8.0
4F	2.2×10^5	3.45×10^8	3.05×10^8	3.05×10^{10}	1.6	1.6	8.0
5F	2.0×10^5	3.40×10^8	3.00×10^8	3.00×10^{10}	1.6	1.6	8.0

(2) frame systems with shear wall or bracing—the fact that the bending-mode deformation contributes mostly to the total deformation of these buildings is not consistent with the assumption of shear-mode deformation in the interpolation technique (Mau and Aruna 1994).

5. Numerical verifications

A five-story building with two-way eccentricity under the 1940 El Centro bi-directional earthquakes (PGA = 0.3 g) is presented to demonstrate the efficiency of the proposed modal parameter identification procedure and mode shape interpolation technique. The system properties of the model structure are given in table 1. The Rayleigh damping with $\xi_1 = \xi_2 = 2\%$ is assumed. Only the second floor and roof acceleration responses are measured. Random noise with a noise-to-signal ratio of 20% was added. The corresponding free decay signatures of the 1F and 5F, extracted using the modified random decrement method, are shown in figures 2(a)–(c). The first three identified modal frequencies, damping ratios and mode shapes versus the true values are given in table 2 and figure 3. It is shown that the proposed method is able to identify the dominant modal parameters accurately even with highly coupled modes and high levels of measurement

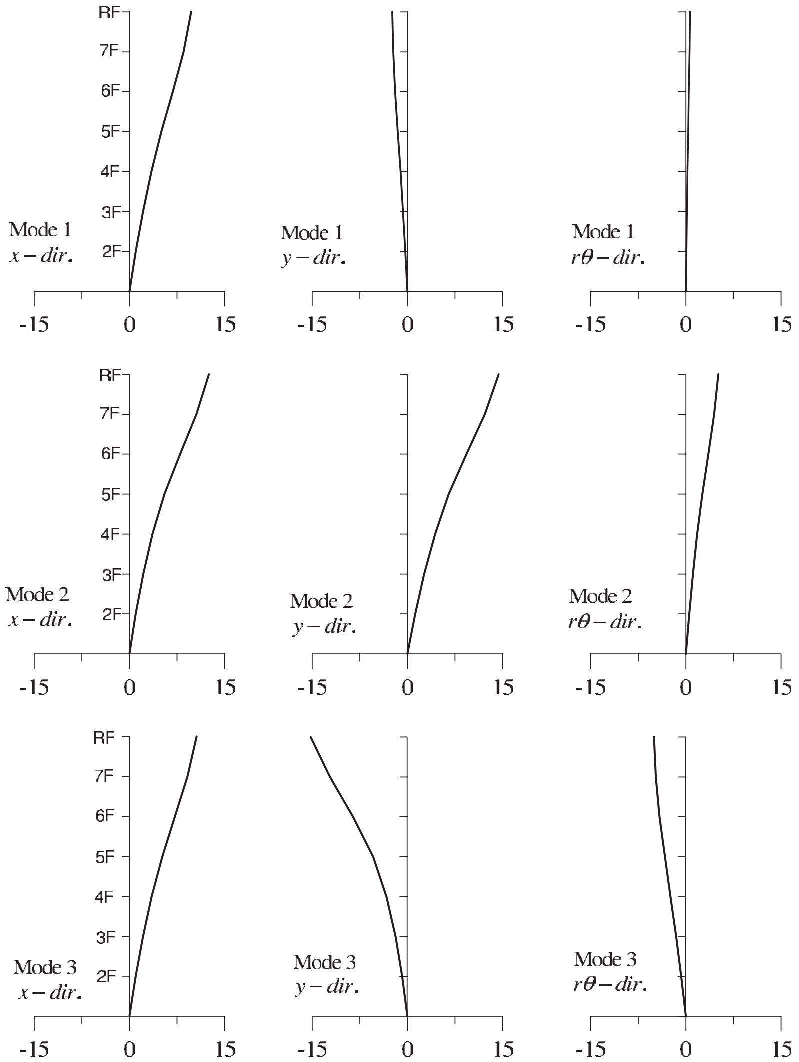

Figure 8. The estimated complete first three mode shapes of the seven-story school building.

Table 2. Identified first three modal frequencies and damping ratios of the five-story building.

	Natural frequency (Hz)	Damping ratio (%)
True	$\begin{bmatrix}1.629\\1.767\\2.267\end{bmatrix}$	$\begin{bmatrix}2.00\\2.00\\2.08\end{bmatrix}$
Estimated	$\begin{bmatrix}1.613\\1.764\\2.278\end{bmatrix}$	$\begin{bmatrix}1.81\\1.91\\1.85\end{bmatrix}$

noise. This identification result is generally adequate for building structures because their total responses are dominated by the first few modes.

In addition, based on the identified modal parameters, the absolute accelerations at each floor were calculated under the bi-directional components of 1940 El Centro earthquake. The peak and root-mean-square (rms) acceleration responses at measured (roof) and unmeasured (4F and 5F) floors are listed in table 3. Since the dominant modal properties are accurately estimated, so are the dynamic responses. There is good agreement between the estimated and true responses.

Table 3. Peak and rms response estimation of the five-story building under 1940 El Centro bi-directional earthquakes.

	Peak response (g)		rms response (g)	
	True	Estimated	True	Estimated
$\ddot{x}_4$	0.802	0.827	0.187	0.173
$\ddot{x}_5$	0.916	0.990	0.222	0.208
$\ddot{x}_{RF}$	1.139	1.075	0.245	0.226
$\ddot{y}_4$	0.917	0.901	0.219	0.224
$\ddot{y}_5$	0.997	1.084	0.262	0.268
$\ddot{y}_{RF}$	1.080	1.178	0.288	0.290
$(r\ddot{\theta})_4$	0.666	0.680	0.132	0.139
$(r\ddot{\theta})_5$	0.739	0.813	0.157	0.167
$(r\ddot{\theta})_{RF}$	0.820	0.882	0.172	0.181

There are some assumptions in using the randomdec signatures of responses only to identify the system parameters, namely the zero-mean stationary Gaussian response excited by a white noise. Therefore, such a technique was originally proposed for ambient vibration. When the randomdec signatures are extracted from the earthquake responses, the

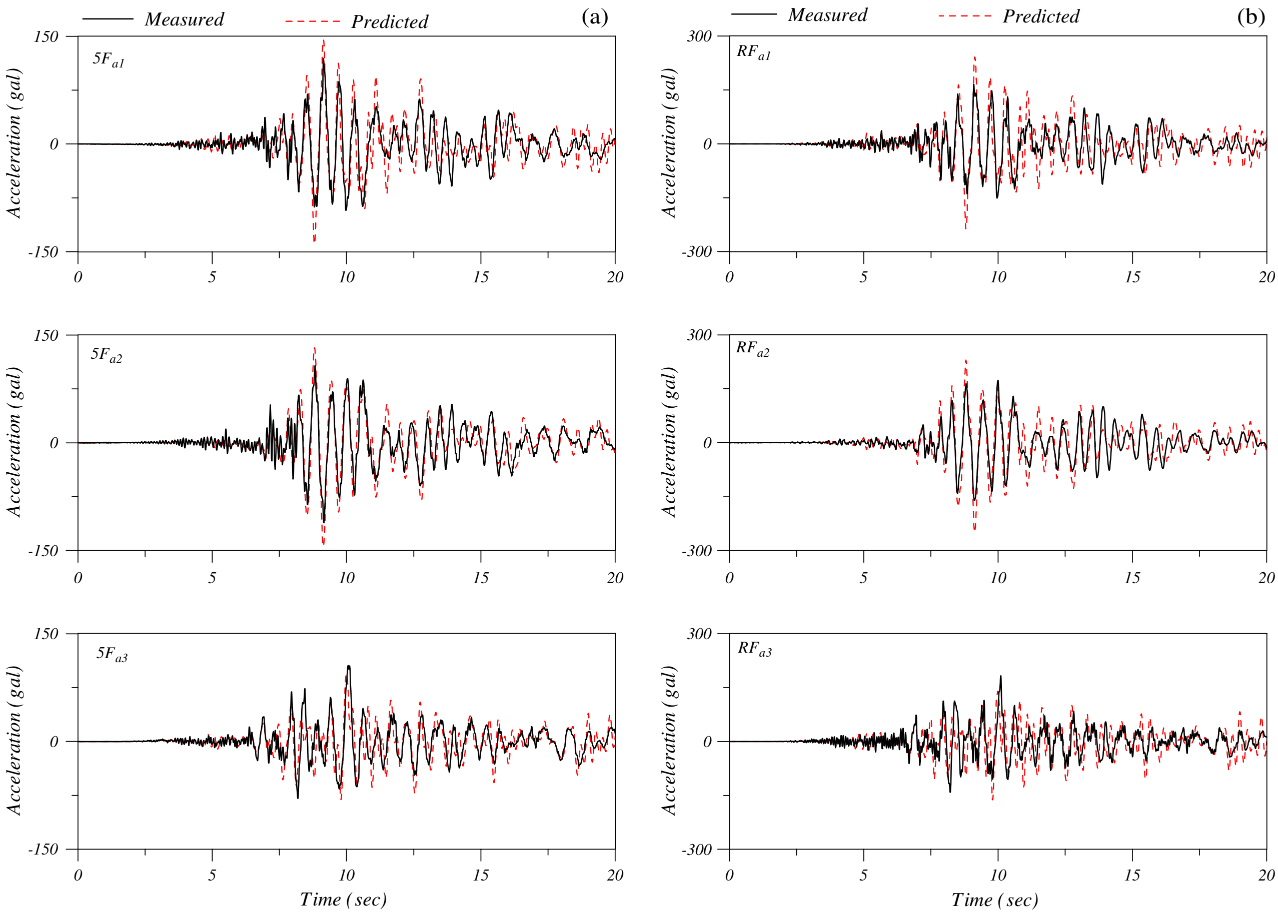

Figure 9. (a) Predicted versus measured acceleration responses at the sensor locations in the fifth floor of the seven-story school building. (b) Predicted versus measured acceleration responses at the sensor locations in the roof level of the seven-story school building.

assumption of a zero-mean process is automatically satisfied after a procedure of baseline correction is performed to the responses. A seismic input is indeed transient, and so are the floor responses. However, the nonstationary part of a response due to the initial conditions decays because of the damping effect, and the stationary behavior is usually approached in the strong-motion response, where the components of a randomdec signature are triggered. Basically, a randomdec signature is the numerical results in estimating a specified conditional expectation through a temporal average scheme. The use of the record length extension technique described in section 2.2 is not only to enhance the stationarity but also to increase the accuracy in estimating the conditional expectation. Such a conditional expectation is proportional to the auto-correlation function of a stationary Gaussian process, as shown in equation (1). For a linear structural system, the response is superposed by all the increments induced by the seismic input ahead, and then the strong-motion response may be assumed to be Gaussian by the central limit theorem.

Equations (5)–(7) are derived from the assumption of a constant power spectral density of the excitation. When the excitation is not a white noise, the three equations are only approximated to the free decay responses. However, the white noise approximation could be applied to a wide-band excitation (Der Kiureghian 1981), and the validity depends on the frequency content of the seismic input. The Fourier amplitudes of the 1940 El Centro bi-directional earthquakes are shown in figures 4(a) and (b). In view of these figures, the seismic inputs are not narrow band and their spectral bandwidths include the first three modal frequencies of the five-story building. Therefore, the dominant modal parameters are accurately identified.

6. Identification of a real RC building

The acceleration response records from a seven-story RC school building in I-Lan, north-eastern Taiwan due to the 1994 Nan-Au earthquake ($M_L = 6.2$) are used to identify the modal parameters and verify the effectiveness of the proposed system identification techniques. This building, used for classrooms and offices, has a one-story basement and central skylight. The rectangular plan is fairly regular. The plan of these floors and a front view are shown in figure 5.

6.1. *Seismometers layout and building model*

Nine seismometers, installed in three floors (the base, fifth and roof floors) with each floor having three seismometers, were used to measure the acceleration responses simultaneously at different locations. The seismometers were located in three combinations in order to measure the dominant modes in each direction. Assume that the building is fixed at the

Table 4. Identified first six modal frequencies and damping ratios of the school building.

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
Natural frequency (Hz)	1.95	2.59	3.70	5.59	6.98	7.15
Damping ratio (%)	5.09	12.43	2.21	7.97	5.35	2.00

base and modeled as a torsionally coupled shear building. The model has three dynamic degrees of freedom at each floor: two translations and one rotation about the mass center. Figure 6 shows the relative position layout of the mass center and seismometers on one of the instrumented floors, where C.M. represents the floor mass center and $a1$, $a2$ and $a3$ the seismometers. The mass center of each floor was located first, then the two translations and one rotation at the instrumented floor were calculated using

$$\ddot{x} = [\ddot{u}_{a2}L1 - (\ddot{u}_{a1} + \ddot{u}_{a2})L2]/L1 \tag{21a}$$

$$\ddot{y} = [\ddot{u}_{a3}L1 - (\ddot{u}_{a1} + \ddot{u}_{a2})L3]/L1 \tag{21b}$$

$$\ddot{\theta} = (\ddot{u}_{a1} + \ddot{u}_{a2})/L1 \tag{21c}$$

where $\ddot{u}_{a1}$, $\ddot{u}_{a2}$ and $\ddot{u}_{a3}$ are the acceleration responses measured from seismometers $a1$, $a2$ and $a3$.

6.2. *Modal parameter estimation*

The free decay acceleration signatures, calculated from the modified random decrement method, at the mass center of the fifth and roof floors in three directions are shown in figures 7(a) and (b). The first six modal frequencies and damping ratios are identified and listed in table 4. The complete first three mode shapes calculated by equation (20) are shown in figure 8. Using the identified first three modal parameters and the acceleration at the fixed base as input excitation, the absolute acceleration responses at the sensor locations in the fifth floor and roof are predicted and compared with actual measurements as shown in figure 9. They are in fairly good agreement in the whole course. The average estimation errors were less than 10%. The accurate structural response estimation indicates that the proposed system identification technique is applicable to real structures using their earthquake response records directly.

However, the discrepancy between the measured and predicted acceleration responses is more obvious than that in the five-story building. The Fourier amplitudes of the base excitations are shown in figures 10(a) and (b). In view of these figures, the identified modal parameters below 2 Hz are convincing, and the identified modal parameters between 2 and 6 Hz are less convincing, compared to the first modal parameter.

7. Conclusions

This study found the analytical general relationship between the reduced randomdec signature and true free decay acceleration response; then, the conventional random decrement method was modified to extract free decay responses using transient acceleration response measurements induced by earthquakes. The ITD technique was employed to evaluate the modal parameters of general torsionally coupled buildings using only few floor acceleration records. Based on the analytical and numerical results from simulation and real measured data, the following conclusions may be made.

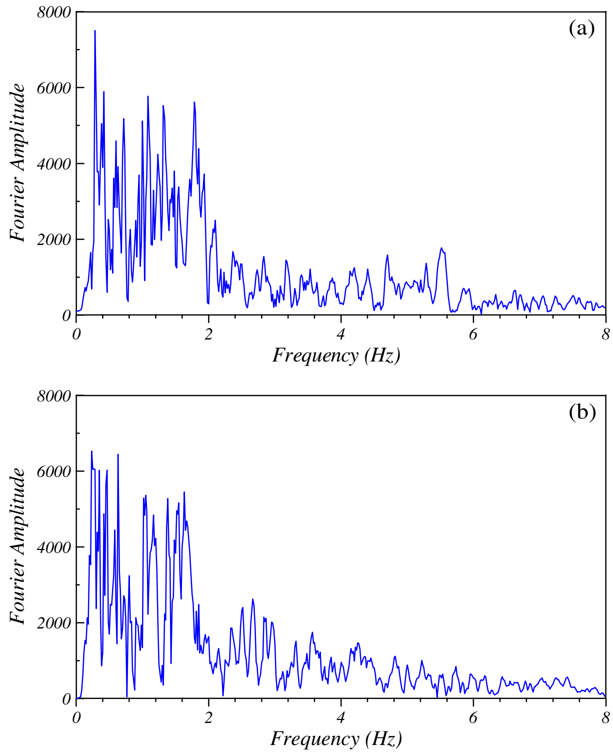

Figure 10. (a) Fourier amplitude spectrum of base acceleration in the EW direction. (b) Fourier amplitude spectrum of base acceleration in the NS direction.

(1) The randomdec acceleration signature is equal to a free decay acceleration response with certain initial displacement and velocity.
(2) The proposed record length extension technique is useful for transient response measurements and thus increases the applicability and accuracy of the random decrement method.
(3) The proposed system identification technique accurately identifies structural dominant modal parameters and responses even with highly coupled modes and high levels of noise contamination.

The small number of response measurements and simple on-line calculations make the proposed method favorable to real implementation.

Acknowledgments

This research was supported in part by the National Science Council of the Republic of China under grant No. NSC 89-2218-E-005-009. This support is greatly appreciated. The authors would like to thank the reviewers for their constructive comments that have improved the quality of this paper.

References

Bedewi N E 1986 The mathematical foundation of the auto and cross-random decrement techniques and the development of a system identification technique for the detection of structural deterioration *PhD Dissertation* Department of Mechanical Engineering, University of Maryland, College Park, USA

Benzoni G and Gentile C 1993 Two approaches to identify equivalent structural models from earthquake responses *Soil Dyn. Earthq. Eng.* **12** 113–25

Caravani P, Watson M L and Thomson W T 1977 Recursive least-square time domain identification of structural parameters *J. Appl. Mech., ASME* **44** 135–40

Cole H A 1971 Method and apparatus for measuring the damping characteristics of a structure *US Patent Specification* 3-620-069

Cole H A 1973 On-line failure detection and damping measurement of aerospace structures by random decrement signatures *NASA CR-2205*

Der Kiureghian A 1981 A response spectrum method for random vibration analysis of MDF systems *Earthq. Eng. Struct. Dyn.* **9** 419–35

Hejal R and Chopra A K 1989 Earthquake analysis of a class of torsionally-coupled buildings *Earthq. Eng. Struct. Dyn.* **18** 305–23

Ibrahim S R 1986 Double least squares approach for use in structural modal identification *J. Spacecr. Rockets, AIAA* **24** 499–503

Ibrahim S R and Mikulcik E C 1977 A method for the direct identification of vibration parameters from the free response *47th Shock and Vibration Bulletin* pp 183–98

Ibrahim S R and Pappa R S 1982 Large modal survey testing using the Ibrahim time domain identification technique *J. Spacecr. Rockets, AIAA* **19** 459–65

Kadakal U and Yüzügüllü Ö 1996 A comparative study on the identification methods for the autoregressive modeling from the ambient vibration records *Soil Dyn. Earthq. Eng.* **15** 45–9

Kan C L and Chopra A K 1977 Elastic earthquake analysis of torsionally-coupled multi-storey buildings *Earthq. Eng. Struct. Dyn.* **5** 395–412

Kozin F and Natke H G 1986 System identification techniques *Struct. Safety* **3** 269–316

Luz E and Wallaschek J 1992 Experimental modal analysis using ambient vibration *Int. J. Analyt. Exp. Modal Anal.* **7** 29–39

Mau S T and Aruna V 1994 Story-drift, shear, and OTM estimation from building seismic records *J. Struct. Eng., ASCE* **120** 3368–85

Mickleborough N C and Pi Y L 1989 System modal identification using free vibration data *Struct. Eng. Earthq. Eng., JSCE* **6** 217s–28s

Shinozuka M and Yun C B 1982 Identification of linear structural dynamical system *J. Eng. Mech. Div., ASCE* **108** 1317–90

Toki K, Sato T and Kiyono J 1989 Identification of structural parameters and input ground motion from response time histories *Struct. Eng. Earthq. Eng., JSCE* **6** 413s–21s

Torkamani M A M and Ahmadi A K 1988 Stiffness identification of a tall building during construction period using ambient tests *Earthq. Eng. Struct. Dyn.* **16** 1177–88

Ueng J M, Lin C C and Lin P L 2000 System identification of torsionally coupled buildings *Comput. Struct.* **74** 667–86

Vandiver J K, Dunwoody A B, Campbell R B and Cook M F 1982 A mathematical basis for the random decrement vibration signature analysis technique *J. Mech. Des.* **104** 307–13

Wang D and Haldar A 1994 Element-level system identification with unknown input *J. Eng. Mech., ASCE* **120** 159–76

Yang J C S and Caldwell D W 1976 The measurement of damping and the detection of damages in structures by the random decrement method *46th Shock and Vibration Bulletin* pp 129–36

Yang J C S, Chen J and Dagalanis N G 1984 Damage detection in offshore structures by the random decrement method *J. Energy Resour. Technol., ASME* **106** 38–42

Zaslavsky Y and Shapira A 1997 Empirical estimates of modal parameters of full scale structures *Eur. Earthq. Eng.* **1** 26–36